

Voice of the Dragon: Harmonies of the Rotating Resonator

M. P. Silverman

[Adapted from a chapter in M. P. Silverman, *Waves and Grains: Reflections on Light and Learning* (Princeton University Press, 1998), initially published, together with my student Mark Cushman, in the *European Journal of Physics* 1989.]

The isle is full of noises,
Sounds and sweet airs that give delight and hurt not.
—William Shakespeare (*The Tempest*)

Summary

A simple, yet unusual, child's toy, in effect a rotating corrugated resonator, generates a wide range of tones by spatial modulation of air flow, and illustrates some of the most basic features of the physics of resonance, waves, and fluids. I encountered the toy when living in Japan with my family and brought some specimens to my laboratory to examine and work out the mechanism of sound production. Although production of sound and electromagnetic waves is quite different in detail, the toy calls to mind an equally unusual light source.

'Hearing' the Light

One of the most remarkable light sources I ever learned of was reported in one of the shortest scientific papers¹ I ever read: less than a single page in *The Physical Review* (a sizable fraction of which was a photograph).

The opening line of the paper

It occurred to one of the authors (EMP) that if an electron passes close to the surface of a metal diffraction grating, moving at right angles to the rulings, the periodic motion of the charge induced on the surface of the grating should give rise to radiation.

reveals in its entirety Purcell's clever idea: charged particles moving at a *constant* velocity can give rise to electromagnetic radiation. Recall that Maxwell's classical theory of electromagnetism predicts that radiation is produced by *accelerating* electric charge. However, passing at speed v close to the corrugated surface of a metal diffraction grating with ruling interval d , individual electrons of an electron beam generate image charges that effectively oscillate normal to the grating surface, as illustrated in Figure 1. In complete conformity with classical electrodynamics, the oscillating image charges should radiate—and a simple Huygens construction predicts the fundamental wavelength to be

$$\lambda = d \left(\frac{c}{v} - \cos \theta \right) \quad (1)$$

¹ S. J. Smith and E. M. Purcell, "Visible Light from Localized Surface Charges Moving Across a Grating", *The Physical Review* **92** (1953) 1069.

where θ is the angle of observation relative to the forward direction of the beam. For a grating with spacing $d = 1.7 \mu\text{m}$ and electrons of energy 300 keV, the predicted radiation falls within the visible spectrum.

Using a small Van de Graaf generator, Smith and Purcell found that, like ‘real’ charges, accelerated image charges do indeed produce light. Viewing their grating from a forward position 10 to 20 degrees from the beam, they saw a ‘sharp, luminous, colored line on the surface of the grating’ whose color varied, as expected, with the observation angle and electron energy.

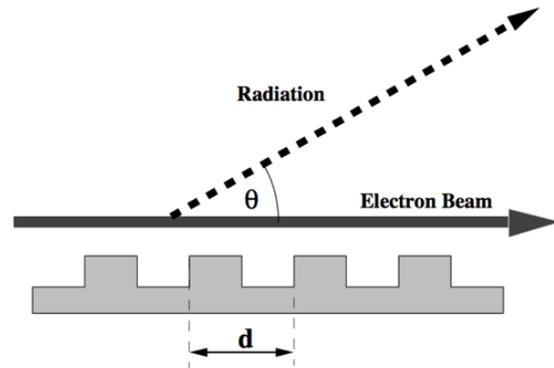


Figure 1: Configuration of the Smith-Purcell light source. An electron beam passing over a grating with spacing d generates an oscillating image charge that radiates.

The authors, as far as I know, never resumed this work—or at least never reported on it again. And when, having discovered this paper some fifteen years or so after its publication, I endeavoured to know more, a dip into the physics literature then available to me produced but a paltry harvest of references. Even today, when on some rare occasion I mention Smith-Purcell (SP) radiation to a colleague, the latter is more likely than not to look at me quizzically. Within the physics community at large, the SP light source still seems to be a fairly well-kept secret.

I had intended at one time to develop a quantum theory of SP radiation, but, as one project led to another, I continually put off the undertaking, eventually dropping the subject altogether. I never returned to it, but many years later I came across a fascinating device that brought this early interest to mind.

One day, while living in Japan as a guest scientist of the Hitachi Advanced Research Laboratory in Tokyo, I emerged with my family from a train station to encounter the eerie strains of a most unearthly symphony. A dozen or so Japanese children, soon joined by my own, were feverishly grabbing long flexible plastic tubes from the stand of a streetside vendor and twirling them above their heads like lariats. The burst of tones that emerged from each musical pipe soared and dropped with rotational speed over what seemed like a good portion of the range of a flute.

Designated *Voice of the Dragon* by a sign in English, each musical tube was basically a right circular cylinder roughly 3 cm in outside diameter and 70 cm in length. The tubes, however, were not smooth; rather, corrugations with a depth of about 1/4 cm and a periodicity of 2 per cm spanned the full length. The overall effect of this loud, wavering, rich-toned chorus of dragon voices left an unforgettable impression.

Despite the outward simplicity of the toy, the details of how it worked were by no means evident. Watching the dragon in action, I puzzled over such questions as the following. Are the characteristic modes of the rotating tube the same as those of a comparable stationary open-ended pipe? Are the modes excited by air rushing past the moving end transversely or through the tube longitudinally? What relation is there between the pitch of a tone and the rate of rotation? Are the corrugations necessary? Can the dynamics of the tube be quantitatively understood, even in part, as a simple application of Bernoulli's principle?

A cursory examination of the tube clarified immediately at least a few points. First, there was a strong air current passing through the rotating tube from the stationary end toward the moving end. By

grasping the tube about 30 cm from one end and rotating the remaining length, I heard the air flow and felt the suction as I progressively covered the stationary end with the palm of the other hand; small bits of tissue paper placed in my palm were sucked up into the tube and discharged from the rotating end. Holding the tube near the middle with both hands and causing the two ends to rotate simultaneously led to no tones, as there was apparently no net air flow through the tube.

Secondly, the corrugations were definitely critical to the production of sound. Inserting into the tube a rolled sheet of paper that formed a smooth inner cylindrical wall eliminated the musical sounds without obstructing the flux of air; one then heard only the whoosh of white noise. Moreover, it did not matter where the paper was inserted; whether at the moving end, stationary end, or middle, the dragon voice remained silent if corrugations were covered. I found, in fact, that merely covering a small part of the last few corrugations of the stationary end with my thumb was sufficient to reduce the sound intensity greatly at any angular frequency of rotation, although it barely effected the volume of flow. The resonant production of sound apparently depended sensitively on the entire structure and was severely attenuated by even small impediments.

Intrigued by the unexpected melodic qualities and behaviour of the dragon voice, and mindful of the unusual physical effects of rotational air flow in other devices I have studied (such as the vortex tube²), I decided to investigate this unusual toy more thoroughly with an undergraduate student upon my return to the U.S. The experiments³, which are simple and instructive, employ such basic procedures as Fourier analysis of wave forms and measurement of air speed and pressure, and easily serve as thought-provoking and interest-generating laboratory projects in courses on general physics, waves, or acoustics. Indeed, these experiments are ideally suited to such purpose, for the musical toy seems to stimulate the curiosity of all who hear its lyrical tones.

Dragon Notes

To answer the questions posed in preceding section, my student and I constructed an apparatus by means of which the musical tube could be rotated in a controlled, reproducible way free from centrifugal distortions. As shown in Figure 2, the corrugated resonator was mounted on a thin wooden slab which was attached to a spoked wheel free to rotate in a vertical plane; a counterweight was fixed at the opposite end of the slab so that the center of mass of the system lay on the axis of the wheel. A motor, whose speed could be varied by a rheostat, was placed in direct contact with the outer rim of the wheel.

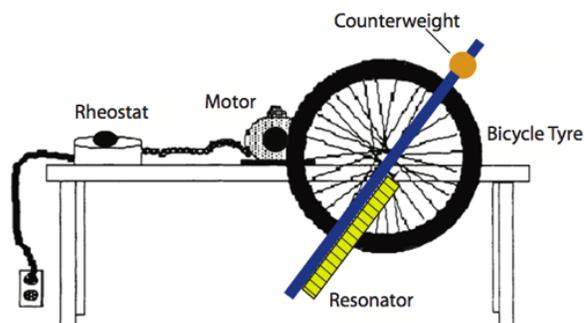


Figure 2: Apparatus for producing distortion-free rotation of the resonator. The tube is fixed to a wooded slab, balanced by a counterweight, and mounted to a wheel whose rim is driven by friction contact with a rheostat-controlled motor.

²M. P. Silverman, 'The Vortex Tube: A Violation of the Second Law?', *Eur. J. Phys.* **3** (1982) 88-92; I have written more comprehensively of the vortex tube and the principles of thermodynamics in my books, (a) *And Yet It Moves: Strange Systems and Subtle Questions in Physics* (Cambridge University Press, 1993) and (b) *A Universe of Atoms, An Atom in the Universe* (Springer, 2002).

³M. P. Silverman and G. M. Cushman, 'Voice of the Dragon: the Rotating Corrugated Resonator', *Eur. J. Phys.* **10** (1989) 298-304.

The first experiments were designed to determine the frequencies of the acoustic resonances of the rotating tube and to compare them with the modes of an identical stationary tube mounted horizontally on a table and activated by the air blast of a blower, such as is commonly used in air track experiments. The air flux into the stationary tube was controlled by varying the distance between the mouth of the tube and the blower.

Assuming the dragon to be a sort of open-ended organ pipe of length L , whose vibrational modes have displacement antinodes (maxima) at the two ends, one would predict the spectrum of mode frequencies to be

$$f_n = \frac{nv_s}{2L} \quad (n = 1, 2, 3\dots) \quad (2)$$

in which v_s is the speed of sound in air. Actually, since the air is moving in and out of the open ends, the effective length L' of the resonating air column is a little longer than L and depends on the radius R of the pipe. For a pipe with two open ends (and $R < \frac{1}{25}$ th of a wavelength) this approximate correction is⁴

$$L' = L + 1.22R \quad (3)$$

Figure 3 illustrates schematically how the musical tones were obtained and processed. A microphone, aligned along the axis of rotation, recorded the tones at the different angular frequencies at which the wheel was spun. These spin rates were measured by means of a stroboscope and counter/timer. The sounds from the dragon were amplified and then Fourier analyzed into their composite frequencies by an Apple IIe computer. *(Most people now reading this essay have probably never seen so ancient a computer model, but it served its purpose well when the experiment was performed in the 1980s.)* In preliminary studies of the frequency spectrum, the computer sampled 2048 points at 50 ms per point; later a higher resolution (and therefore slower) program with a 200 ms sampling time per point was employed for more precise work.

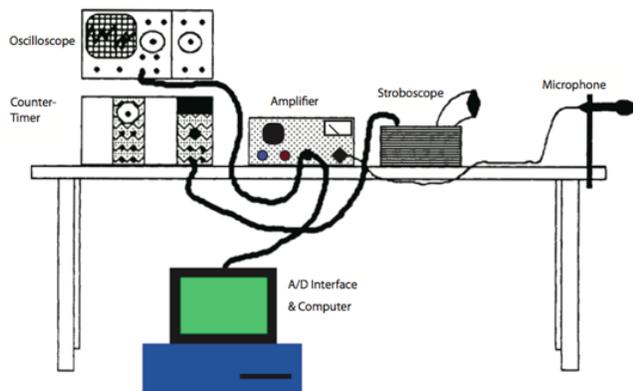


Figure 3: Apparatus for recording sound amplitude against time plots of the acoustic resonances. The sound traces are picked up by a microphone, amplified, displayed on an oscilloscope, and sent to a computer for Fourier analysis into composite frequencies. The rotation rate is determined by means of a stroboscope and counter/timer.

An example of the temporal recording of sound intensity for the $n = 6$ acoustic mode is shown in Figure 4. Theoretically—i.e. according to Eqs. (2) and (3)—the frequency of this mode should be 1350 Hz. The weak amplitude modulation which one sees in the figure could be produced by a non-uniform rotation of the motor shaft, or small distortion of the wheel, or offset of the microphone from precise alignment along the axis of rotation. Such modulation can be useful by providing an independent confirmation of the rotation rate. The recording in the figure was intentionally made with a slightly offset microphone. However, the data that were actually used to interpret the mechanism of sound production were obtained with a uniformly rotating motor, undamaged wheel, and centrally placed microphone.

⁴L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954) 133.

The amplitude (in the musical sense, i.e. sound intensity) against time plots of each mode were recorded at a spin rate just slightly beyond that for threshold production of the tone. Increasing the rate shifted the resonance frequency upward in accordance with expected intramode frequency shifts characteristic of organ pipes; a given mode would be extinguished by about 10 Hz beyond threshold. Further increase in rotation rate then produced low-level white noise until the sudden onset of the next mode. For the stationary tube excited by a blower over whose air stream there was less precise control, the intermode transitions occurred smoothly and continuously, rather than discretely, with increase in air flux.

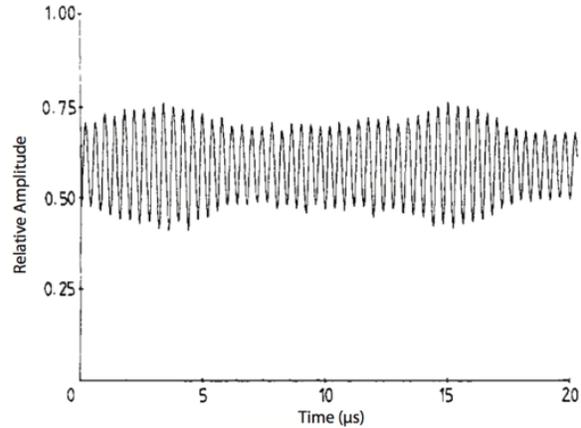


Figure 4: Sound amplitude against time for the mode $n = 6$ as recorded by a microphone slightly offset from the axis of rotation.

A composite spectrum of resonance frequencies, obtained from Fourier analysis of sound variations like that of Figure 4, is illustrated in Figure 5 which shows the modes $n = 2, 3 \dots 6$ for the rotating tube. The fundamental ($n = 1$) mode is absent, for it was found difficult to produce and sustain this mode whether the tube was rotated by hand or by motor.

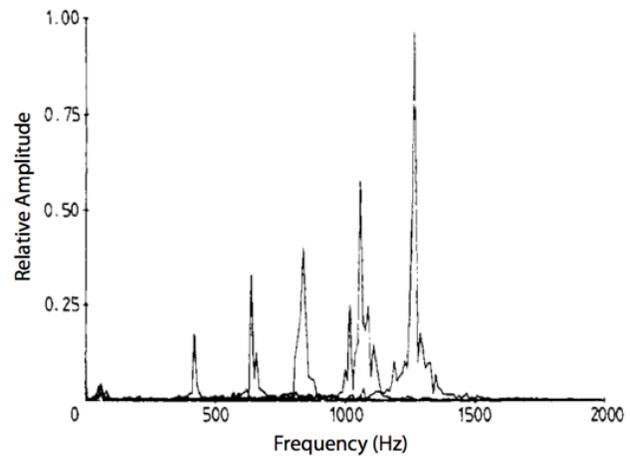


Figure 5: Frequency spectrum of the rotating tube for modes $n = 2-6$ as obtained by Fourier analysis of temporal plots like that of Figure 14.4.

The resonance data ('dragon notes') for a tube of length $L = 74$ cm, outside radius $r = 1.5$ cm, and corrugation spacing $d = 0.6$ cm are summarized in Table 1. At the ambient temperature of approximately 16°C , the speed of sound in air was $v_s = 342$ m/s. The first column of the table records the theoretical resonance frequencies of an ideal open-ended organ pipe (with end correction applied); for the stated geometry the fundamental is 225 Hz. In the second and third columns are the corresponding experimental frequencies of the stationary and rotating tubes, respectively, measured with an estimated precision of ± 3 Hz. The modal frequencies of the stationary tube were about 3-4% below the theoretical values, and the frequencies of the rotating tube were approximately 3% below those of the stationary tube. That the resonance frequencies of the dragon voice, whether stationary or rotating, are lower than the comparable theoretical values of a smooth, open-ended pipe results, I suspect, from the corrugations which give the tube an effective length longer than L' . The reduction in pitch of the modes of the rotating tube below comparable values of the stationary tube seems to be a consequence of the lower threshold air speeds through the rotating tube, as indicated in the fifth and sixth columns. I will discuss the determination of air speed and pressure difference (across the tube) shortly.

The initial experiments established that, for purposes of modeling, it was not unreasonable to regard the dragon as a kind of rotating organ pipe (although one whose length was a bit slippery to pin down precisely). But what makes the dragon sing?

Table 1: Threshold Mode Frequencies and Air Speeds					
Mode n	Threshold Mode Frequencies (± 3 Hz)			Air Speeds (± 0.3 m/s)	
	Theory^a (Hz)	Stationary Tube (Hz)	Rotating Tube (Hz)	Stationary Tube (m/s)	Rotating Tube (m/s)
1	225	—	—	—	—
2	450	432	419	6.3	5.4
3	675	646	632	9.23	8.2
4	900	861	839	11.8	10.7
5	1125	1083	1051	14.9	13.3
6	1350	1310	1263	18.9	17.1

^a Eqs. (2) and (3) with $v_s = 340$ m/s, $L = 0.74$ m, $R = 0.15$ m

A Bernoulli Pump

To probe the dynamics of the dragon voice further, it is necessary to examine different aspects of the air flow through the tube. Despite the outward simplicity of construction, the actual pattern of air flow is complex, and it is useful to consider separately the hierarchy of scales at which the flow takes place. The largest-scale flow, as perceived in the rest frame of the tube, is vortical, centered on the pivoted end and moving perpendicular to the axis of the tube with a tangential air speed ideally given by

$$V(s) = 2\pi u s \quad (4)$$

where u is the rotation rate (in rev/s) and s is the distance (ranging from 0 to L) along the tube. Next in scale is an axial flow through the tube produced by the rotationally induced pressure difference between the ends. Finally, at the smallest scale is the internal flow over the corrugations, which becomes increasingly turbulent as the rotational rate and mode number increase.

The objective here is not to provide a rigorous hydrodynamical analysis of the dragon voice, but rather to understand some of the basic features of its behaviour at as basic a level as possible, so that anyone being introduced to general physics can appreciate it.

Let us consider first the question of how the resonant pressure difference across the length of the tube and the rotation rate are related. For a smooth, incompressible mesoscale flow of fluid with mass density ρ Bernoulli's principle (with neglect of gravity) leads to the relation

$$p(s) + \frac{1}{2} \rho \left[v(s)^2 + 4\pi^2 u^2 s^2 \right] = \text{constant} \quad (5)$$

in which $p(s)$ is the pressure and $v(s)$ is the axial velocity at location s along a streamline. Air, of course, is a compressible fluid, but it is assumed that compression can be neglected for the range of gentle pressure differences created by tube rotation. Under the same conditions, therefore, the conservation of mass flow through a tube of constant cross section yields a uniform axial velocity within the tube, i.e. $v(s) = \text{constant}$ for $0 \leq s \leq L$. It then follows that the pressure difference Δp between the stationary and rotating ends is given by

$$\Delta p = p(0) - p(L) = 2\pi^2 \rho L^2 u^2. \quad (6)$$

Eq. (6) is consistent with one of my earliest observations, namely that the pressure at the rotating end is lower than that at the stationary end (which accounts for why the bits of paper were sucked out of my palm). The dragon is a kind of ‘Bernoulli pump’.

Consider next the question of how the resonant tones and the rate of rotation are related. Here the microscale flow plays a critical role. Moving past the corrugations, the air is perturbed at a frequency proportional to the axial air speed and inversely proportional to the corrugation spacing d . Let us assume that when this frequency coincides with a resonant frequency the resulting sound is amplified by the tube. Thus the resonant frequency f_n of a given mode n may be related to the axial air speed v_n of the same mode by an expression of the form

$$f_n = a^{-1} \frac{v_n}{d} \quad (7)$$

where a is a proportionality constant. In the ideal case of frictionless flow, application of Eq. (5) at the stationary end of the tube and at a point ($s = L_+$) just outside the rotating end (where $p(0) = p(L_+)$ and the streamline is assumed to be perpendicular to the tube) leads to a uniform axial speed

$$v_n = 2\pi u_n L = V_n(L) \quad (8)$$

which is equal to the tangential speed at the rotating end. The flow, however, is not perfectly frictionless, or there would be no sound. It seems reasonable to assume, however, that v_n is proportional to $V_n(L)$ in which case one can write

$$v_n = b^{-1} 2\pi u_n L \quad (9)$$

where b is another proportionality constant.

Combining Eqs. (7) and (9) leads to the expression

$$f_n = (ab)^{-1} \left(\frac{2\pi L}{d} \right) u_n \quad (10)$$

which predicts a linear proportionality between the threshold acoustic and rotational frequencies.

To test the preceding model of the dragon voice, in particular the relationships

(a) $V(L) = 2\pi uL$ [Eq. (4)]	(b) $\frac{\text{axial air speed}}{\text{acoustic frequency}} = \text{constant}$ [Eq. (7)]
(c) $\frac{\text{rotation rate}}{\text{axial air speed}} = \text{constant}'$ [Eq. (9)]	(d) $\frac{\text{acoustic frequency}}{\text{rotation rate}} = \text{constant}''$ [Eq. (10)]

my student and I measured the resonant acoustic frequency, rotation rate, air pressure difference across the tube, tangential air speed at the rotating end, and axial air speed at the fixed end for the modes $n = 2$ through $n = 6$.

The determination of air pressure and tangential air speed at the rotating end—which, by $n = 6$, was whipping round at over 20 m/s—posed at first a difficult task, but was eventually accomplished in the following way, as illustrated in Figure 6. A narrow-bore rubber sampling tube was mounted along the outside of the corrugated resonator to sample the air stream at the rotating end. Near the stationary end, the sampling tube was connected to an air-tight swivel joint (Aeroquip Type E-75)—a spring-loaded device with high-density carbon seal designed primarily for high-speed cooling rolls on printing presses, paper converters, plastic sheeting machinery and metal foil processing equipment. But it served admirably on the dragon. The static end of the swivel joint was connected to another segment of rubber tubing that attached directly to an inclined tube manometer filled with a non-hygroscopic fluid capable of measuring pressures up to 300 Pa with a resolution of 2 Pa. The frame of the manometer was calibrated for both pressure (in Pa) and air speed (in m/s), the two scales being related by $\text{air speed} = \sqrt{2 \times (\text{pressure}/\text{mass density of air})}$.

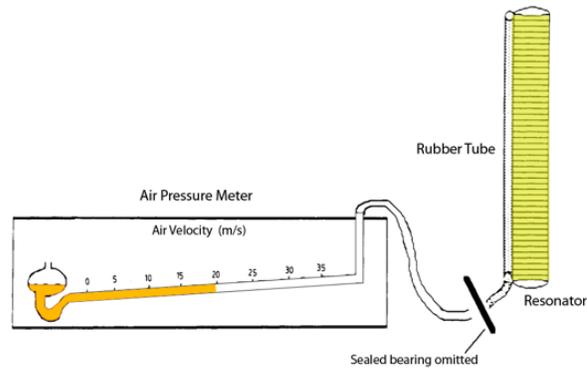


Figure 6: Schematic diagram of Rechenberg pressure tube and manometer for recording pressures and speeds.

The static end of the swivel joint was connected to another segment of rubber tubing that attached directly to an inclined tube manometer filled with a non-hygroscopic fluid capable of measuring pressures up to 300 Pa with a resolution of 2 Pa. The frame of the manometer was calibrated for both pressure (in Pa) and air speed (in m/s), the two scales being related by $\text{air speed} = \sqrt{2 \times (\text{pressure}/\text{mass density of air})}$.

To measure the axial air speed for each mode, a short sampling tube attached to the swivel joint was inserted a few centimeters inside the fixed end of the resonator where the macro-scale tangential flow should ideally be null. Since the inserted probe largely extinguished the resonances, and one could not hear the dragon voice while making measurements of internal air speed, the rotation rate for each mode was set close to the threshold value previously established in the determination of modal frequencies.

The outcome of all these measurements of ‘dragon breath’ are recorded in the following three tables.

Table 2 summarizes the pressure measurements as a function of rate of rotation. The agreement of these experimental results with the predictions of Eq. (6) is very good. For modes $n = 2$ and 3, with rotation rates under 3 rev/s, the theoretical pressure differences are within the estimated experimental uncertainty of pressure measurement. As one might have expected, however, higher rotation rates led to greater turbulence and therefore to departures from streamline flow. Nevertheless, for the five modes accessible to study, the fractional deviation between theory and experiment was under 7%.

Table 3 compares the measurements of tangential air speed at the rotating end (i.e. effectively the end pressure relative to atmospheric pressure) with the theoretically expected speed based on tube length and rotation rate. Agreement is again excellent at low speeds with a fractional deviation of less than 1%; for modes $n = 5$ and 6 the fractional deviation increases to 2-3% as a result of increased turbulence. The measurement of tangential air speed, or pressure, provides a third method (in addition to stroboscope and Doppler effect) of determining the rotation rate.

Last, Table 4 recapitulates the results of pitch, rotation rate, and axial air speed measurements. The fifth column shows the effective constancy of the ratio of mode frequency to axial air speed; the axial air speed is found to be about 80% the tangential air speed for all modes sampled. Similarly, the sixth column confirms the effective constancy of the ratio of axial air speed to rotation rate, and the seventh column establishes the effective constancy of the ratio of mode frequency to rotational frequency. (Any two of these three columns are independent.)

Mode n	Resonance Frequency (Hz)	Rotation Rate (Rev/s)	Pressure Difference (± 1 Pa)		
			Experiment (Pa)	Theory ^a (Pa)	Fractional Deviation ^b (%)
2	419	1.47	28	28.5	0.2
3	632	2.33	73	71.6	-2.0
4	839	3.14	134	130.0	-3.1
5	1051	3.75	198	185.5	-6.7
6	1263	4.40	266	255.5	-4.1

^a Eq. (6) with mass density of air $\rho = 1.22 \text{ kg/m}^3$

^b $100 \times (\text{Theory} - \text{Experiment})/\text{Theory}$

Mode n	Pressure Difference (Pa)	Rotation Rate (Rev/s)	V(L) ^a Experiment (m/s)	V(L) ^b Theory (m/s)	Fractional Deviation ^c (%)
2	28	1.47	6.8	6.83	0.4
3	73	2.33	10.9	10.83	-0.7
4	134	3.14	14.8	14.6	-1.4
5	198	3.75	18	17.44	-3.2
6	266	4.4	20.9	20.46	-2.2

^a $V(L) = \sqrt{2\Delta p/\rho} = 1.28\sqrt{\Delta p}$

^c $100 \times (\text{Theory} - \text{Experiment})/\text{Theory}$

^b $V(L) = 2\pi u_n L = 4.65u_n$

Mode n	Resonance Frequency f (Hz)	Rotation Rate u (Rev/s)	Axial Air Speed (m/s)	a ^a	b ^b	ab ^c
2	419	1.47	5.4	2.1	1.3	2.7
3	632	2.33	8.2	2.2	1.3	2.9
4	839	3.14	10.7	2.1	1.4	2.9
5	1051	3.75	13.3	2.1	1.3	2.8
6	1263	4.40	17.1	2.3	1.2	2.8

^a Eq. (7) with $a = v_n/f_n d$

^c Eq.(10) with $ab = (2\pi L/d)(u_n/f_n)$

^b Eq.(9) with $b = 2\pi u_n L/v_n$

The results, it would seem, are gratifyingly consistent with the hypotheses underlying the proposed mechanism of the dragon voice.

The Dragon Causes Friction

The investigations outlined in the previous section were long completed—and indeed I had already returned from a second invited stay in Japan—when a letter arrived one day from a physicist in

California. Unknown to my student and me, he had, himself, experimented on the rotating resonator some sixteen years earlier⁵ and enjoyed our paper about the dragon voice—although he would have enjoyed it more had he been referenced, which of course was understandable. I replied politely, expressing our regret and pointing out that sixteen years was, after all, a long time and that most scientific papers, once published, ordinarily fade from communal awareness in a much briefer period than that.

The author had arrived at much the same conclusions as my student and I had, although his experimental methods were simpler and devilishly clever. To ascertain the longitudinal air speed at which the dragon (then marketed in the U.S. as a ‘Hummer’) produced different tones, he attached it to his car and took it for a scientific joy ride, exciting the 11th harmonic at approximately 80 miles per hour (129 km/hour). Whether the California Highway Patrol intervened or not, I cannot say, but he eventually devised an alternative procedure. One end of the corrugated tube was inserted into the bottom of a cylindrical wastebasket, and the inverted basket was pressed into a large tub of water. With the water as a piston to force air (assumed incompressible) through the resonator, he could readily determine the air flow through the latter from the rate of linear displacement of the basket in water. The author had, in fact, made several musical instruments in this manner and enclosed with his letter a photograph from a local newspaper with the title ‘Professor Blows His Own Horn’, which somehow seemed quite fitting under the circumstances.

With other projects at hand and a lengthy stay in New Zealand impending, thoughts of the dragon again receded from my mind as quickly as before. From time to time, ever since the dragon paper was first published, letters from interested readers continued to arrive from around the globe, although most merely requested reprints or posed simple questions that did not require much thought on my part. This was fortunate, for the apparatus had long ago been dismantled. Then, one day, there arrived in the mail a request for help from a Gymnasium student in Germany that began:

*Dear Mr. Silverman,
I am a German student and I have to write my research paper about the rotating corrugated resonator...Now I have two problems with it and it would be very pleasant of you, if you could send me an answer.*

The first problem was not difficult, but the second was puzzling indeed:

...you found out that $f_n = a^{-1}v_n/d$ and that the value of the proportionality constant a is more than 2 [actually, it was just about 2], but another article...shows that $a = 1$ Is there anything wrong about the consideration or is it perhaps your measurement?

The article to which the student referred was none other than that of the physicist who had written me some six years earlier. Having seen from a rapid perusal of the text that our conclusions were similar, and intrigued at the time more with his methods and musical instruments, I had not noted that there was, in fact, this curious discrepancy. Was the resonant axial air speed actually *equal* to, or merely *proportional* to, the product of acoustic frequency and corrugation spacing?

Although I was in the midst of other projects, which I was reluctant to interrupt, I could not put the matter aside. It was not that the distinction between the two possibilities was of earth-shaking

⁵F. S. Crawford, "Singing Corrugated Pipes", *Am. J. Phys.* **42** (1973) 278-288.

importance, but rather the many years of doing physics had taught me that it is often in pursuit of small departures from expectation that one learns interesting things. And so it was in this case as well.

Stimulated by the student's letter, I considered the model of the dragon voice once more, and thereby rediscovered what Isaac Newton had already known several centuries ago: the consequences of friction. By means of simple flow experiments, Newton reached two conclusions fundamental to nearly everything that pertains to the mechanics of real fluids. First, a fluid does not slide along a solid, but adheres to it; thus, both the tangential and normal components of flow velocity vanish at a bounding surface. Second, the frictional force (shear stress) at any internal point of the flow is proportional to the velocity gradient (flow shear) normal to the flow. The proportionality constant μ , the coefficient of shear viscosity, is independent of the flow geometry.

That the flow through the corrugated resonator entailed friction was clear to me from the outset of the investigations, and I in fact commented on this point in the published dragon-voice paper. I had not, however, taken explicit account of the following glaring implication: If the air flow is maximum at the center of the tube and vanishes at the cylindrical boundary, then clearly there must be a radial velocity gradient; to have assumed a uniform flow was not strictly correct.

Figure 7 shows the forces acting on a designated cylindrical volume of air of radius r and length z moving at constant speed v through a cylindrical tube of radius R . Since the volume of air is not accelerating, the force from unequal pressures p_1, p_2 on the circular ends is balanced by the force of the shear stress τ along the sides, and the condition of dynamic equilibrium becomes

$$\pi r^2 \frac{dp}{dz} + 2\pi r \tau = 0 \quad (11)$$

in which Newton's law of fluid friction, applied to an axially symmetric system, is simply

$$\tau = \mu \frac{dv}{dr} . \quad (12)$$

Combining Eqs. (11) and (12) results in a first-order differential equation for the longitudinal speed which can be readily integrated to yield a solution of the form

$$v(r) = v_m \left(1 - \frac{r^2}{R^2} \right) \quad (13)$$

with v_m the maximum speed along the tube axis ($r = 0$).

Now in the experiments with my student, the narrow-bore sampling tube measured in effect the axial air speed v_m . On the other hand, the speed inferred from riding with the dragon attached to a car, or pumping it with a water piston, corresponded to the *average* speed

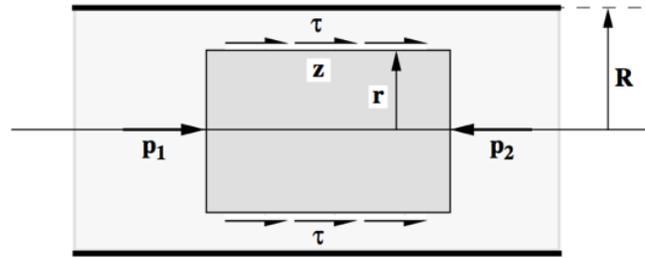


Figure 7: Viscous flow through a cylindrical tube. When the flow rate is constant, the difference in pressure forces is balanced by the frictional shear force.

$$\bar{v} = \frac{1}{\pi R^2} \int_0^R v(r) 2\pi r dr . \quad (14)$$

Performing the integration of Eq. (14) with the velocity law (13) leads to the interesting result $\bar{v} = \frac{1}{2} v_m$. Thus, upon replacing v_n in Eq. (7) by the equivalent expression $2\bar{v}_n$ and substituting the empirically determined value $a = 2$, one obtains the result $f_n = \bar{v}_n/d$. The two factors of 2 have canceled—and the discrepancy has vanished.

Although the preceding analysis neglects the corrugations of the resonator and rigorously applies only to a smooth tube, it incorporates the primary effect of friction which is to generate a nonuniform flow. Presumably the velocity law deriving from a more comprehensive model would resemble that of Eq. (13) except in the immediate vicinity of the corrugated walls. For what it is worth, I note that if the power of the term r/R were some general value k , then Eq. (14) would lead to a mean speed $\bar{v} = v_m \left(1 - \frac{2}{k+2}\right)$ which yields $\bar{v} = \frac{1}{3} v_m$ for $k = 1$ and $\bar{v} = \frac{3}{5} v_m$ for $k = 3$. As k increases, the mean speed approaches the maximum axial speed. Thus, for relatively low values of k the mean and maximum axial air speeds are sensitive to the effects of friction and their measurement can be useful in refining further the theoretical model.

In any event, I believe the dragon is now reasonably tamed. At the most basic level, it can be understood as an open-ended organ pipe blown by an air stream pumped by a rotationally induced pressure difference following Bernoulli's principle. Resonant amplification of sound is initiated at a rotation rate for which the axial air flow is perturbed by corrugations at a frequency corresponding to one of the characteristic frequencies of the tube. It is worth noting that the relationship $f_n = \bar{v}_n/d$, with mean air speed replaced by electron speed, is in fact the same expression that results from the nonrelativistic limit $v/c \rightarrow 0$ of Eq. (1) for the Smith-Purcell light source.

As for the German student, I heard from him again a year later and am happy to say he did very well:

Dear Mr. Silverman,

...Now I am in the gap between 'Gymnasium' and university and finally have time to thank you for your help and send you my research paper. Indeed your letter was a great help...Unfortunately I made a mistake...and used Bernoulli's law in a wrong way. Although my teacher didn't understand this law either, he recognized the mistake, which lead to the result that I 'only' got 14 out of 15 points, which was still one of the best results...Anyway, I thank you very much for answering my letter and giving me some tips for my work. Enclosed there is a shortened version of my work (the longer version for my teacher contains some babble just to make it longer) I used in a special contest called 'Jugend forscht' in which I won 100 DM.

In its charming simplicity, the letter reminded me in *various* ways how much of science education transcends national boundaries!

Those with a questing spirit could undoubtedly find in the Voice of the Dragon further intricacies to explore that will enhance their understanding and enjoyment of physics. Someday, perhaps prompted by other letters, I may return to the subject myself. But for now I have had enough of dragons.