The Not-Quite-So-Simple Operation of 'The World's Simplest Motor'

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What wondrous new machines have late been spinning.
—Lord George Gordon Byron, *Don Juan* (1819)

Summary

A simple motor, which I discovered on YouTube, can be constructed in a few minutes with an AA battery, a small, but strong, cylindrical rare-earth magnet, and a short (~16 cm) length of narrow-gauge copper wire. Shaped appropriately, the wire will rotate spontaneously about the battery. I discuss quantitatively the salient electro-mechanical principles underlying how this motor works and address a number of fundamental, but subtle, conceptual issues that this device raises.

The World's Simplest Motor

One can find almost anything on YouTube, including novel physics demonstrations. It was there, in fact, where I first saw what was referred to as 'the simplest motor in the world'[1]. Subsequently I made my own model, and it was indeed simple. I placed a 1.5 volt AA battery on a small toroidal neodymium magnet pedestal, bent a copper wire into a more-or-less rectangular frame with a "pinch" at the center of the top to serve as a pivot, and placed the frame on the top terminal of the battery with the lower ends of the frame shaped so as to slide along the cylindrical surface of the magnet, as schematically illustrated in Figure 1. The wire frame immediately began spinning around the axis of the battery. At the first few trials, the frame spun for a few moments, then gyrated off the battery. However, when adjusted better, the frame spun for long periods of time at a uniform angular frequency. Watching it for the first time or even after many times, I was fascinated by how so simple a construction gave rise to so striking an effect.

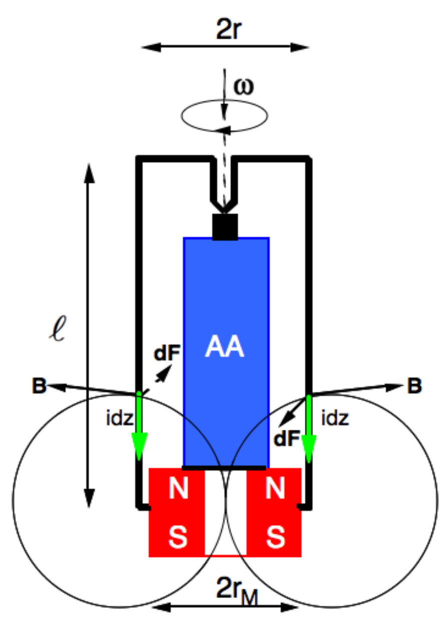

Figure 1: Schematic diagram of the 'world's simplest motor', comprising an AA battery, cylindrical neodymium magnet, and a wire frame serving as rotor. The symbols designating geometrical and electrical quantities are explained in the text.

Although simple to make, the operation of this motor raises questions that are not necessarily simple at all as they entail various fundamental principles of both electromagnetism and mechanics. Why does the frame spin? In which sense (clockwise or counterclockwise) does it spin?

[1] 'Simplest Motor of the World', http://www.youtube.com/watch?v=zOdboRYf1hM.

Why does the rotational angular frequency reach a *steady state*? What relation describes *how* the spin builds up to a steady state? Why isn't the battery *shorted out* when its terminals are connected by a copper wire whose electrical resistance is virtually negligible?

And what about Faraday's law of induction—does it apply here? In textbook diagrams showing the operation of a standard motor as the rotation of a flat wire loop between the poles of a dipole magnet, the axis of the loop is perpendicular to the magnetic field lines, with the consequence that there is a time-varying magnetic flux through the surface of the loop. The toroidal neodymium magnet is also a dipole, but the construction of the 'simplest motor' is such that the dipole axis ideally coincides with the axis of rotation so that the magnetic field lines are cylindrically symmetric about the battery and always lie *in* the plane of the wire frame irrespective of its angular orientation. [Recall that the magnetic field at any point **r**, except at the origin $r = 0$, due to a dipole moment **m** is given by the expression $\mathbf{B} = \frac{\mu_0}{4\pi r^5}\left[3\mathbf{r}(\mathbf{r}\cdot\mathbf{m}) - r^2\mathbf{m}\right]$ where μ_0 is the permeability of free space.] So...can there be any variation in magnetic flux *through* the frame as the frame spins? And if not, is Faraday's law not applicable here?

Simple toys and curious phenomena readily observed at home often provide an attention-arresting starting point for exploration of fundamental physical principles in the classroom.[2] I now address these questions.

Forces and Torques—A First Look

Figure 2 shows a schematic diagram of the circuit corresponding to the physical model in Figure 1 in the ideal case of a perfectly symmetric frame (length ℓ, radius r) spinning about the axis of a battery of potential V and internal resistance R_i oriented with positive terminal up. The cylindrical magnetic pedestal is oriented with the north magnetic pole up. The poles of the magnet were not marked. To determine the polarity, I used a compass needle. Recall that the north pole of a magnet is the pole that *points* north. Thus, the pole to which the north end of a compass needle is attracted is a south pole.

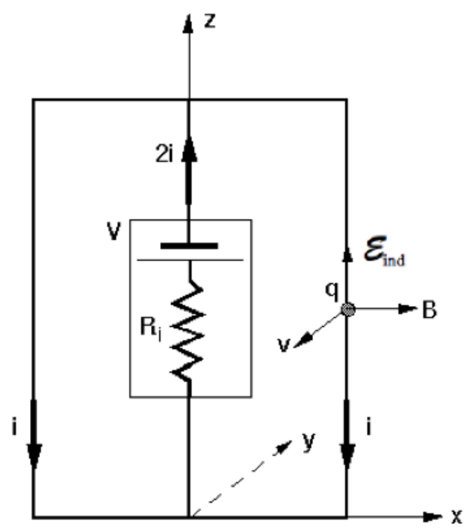

Figure 2: Schematic diagram of the circuit corresponding to the physical motor in Figure 1. The battery is represented as a source of potential V and associated internal resistance R_i. A counter-directed emf is induced in the frame spinning in the presence of the dipole magnetic field. The induced emf would drive a positive test charge q in a sense opposite to that of the current i driven by the battery.

The circuit is pictured in its rest frame where I have chosen a coordinate system with horizontal x-axis, vertical z-axis, and the y-axis directed into the page. A current $2i$ is drawn from the positive terminal, splits equally so that i passes through each arm of the frame, then recombines at the negative terminal. The internal resistance of a battery arises from both electronic and ionic contributions and generally varies with battery age,

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[2] M. P. Silverman, *And Yet It Moves: Strange Systems and Subtle Questions in Physics* (Cambridge University Press, Cambridge, 1993)

load, and environmental conditions.[3] For a fresh battery, it can be in the vicinity of a few tenths of an ohm to an ohm. In contrast, the electrical resistance of a short length (~10 cm), 20-gauge (diameter ~ 1 mm) copper wire of resistivity ~17 $\text{n}\Omega\text{m}$ is a few thousandths of an ohm. I will neglect hereafter the resistance of the copper wire.

The contribution **dF** to the magnetic force on a directed segment **dl** bearing current *i* is $\mathbf{dF} = i\mathbf{dl} \times \mathbf{B}$. The dipole field **B** has components B_x and B_z, but only the horizontal component B_x contributes to the force on a vertical current segment idz. With the north pole of the neodymium magnet oriented upward, B_x is directed along +*x* at the right wire and along –*x* at the left wire (as shown in Figure 1). From the right-hand rule associated with the cross product in the force law, it follows that the force on the right wire is directed along –*y*, and the force on the left wire is directed along +*y*. Since the two forces do not act at the same point, they produce a torque (directed along –*z*) causing the frame to rotate clockwise (cw) as viewed from above the apparatus. In the motor that I made, the horizontal current segments of length *r* are sufficiently short that I will neglect forces on or by them. However, it is easy to show from the above force law that the right and left horizontal currents likewise experience forces into and out of the page, respectively.

I will determine the magnitude of the torque shortly, but first let us consider the potentials within the circuit.

Induced EMF

According to Kirchhoff's voltage law, the potential changes around a closed loop must sum to zero, an expression of the law of conservation of energy in the electrical domain. Kirchhoff's voltage law is not valid, however, if there is a time-varying magnetic flux through the circuit as would be the case, for example, if the circuit in Figure 2 encompassed an ac current-bearing solenoid oriented perpendicular to the page. This exclusion does not apply to the circuit of Figure 2 because of the symmetrical distribution of the magnetic field lines, which have no component perpendicular to the surface enclosed by the circuit in its rest frame. It would be a mistake to think, however, that the only two contributions to this sum in the present case are the rise in potential *V* between the negative and positive terminals of the battery and the corresponding drop in potential $2iR_{\text{i}}$.

Consider a positive test charge *q* located in the right vertical wire as shown in Figure 2. The instantaneous velocity **v** of that charge is directed out of the page with a magnitude $\text{v} = \omega r$, since the frame is rotating cw with angular frequency ω. (In reality, the charge may also have a drift velocity within the wire, but this can be neglected compared with the macroscopic motion of the wire carrying the charge.) Thus, the charge is subjected to a Lorentz magnetic force $\mathbf{F}_{\text{q}} = q\mathbf{v} \times \mathbf{B}$, directed *upward* along the vertical wire, thereby giving rise to an induced emf (i.e. work done per unit of charge transported over a specified distance)

$$\mathcal{E}_{ind} = \int_0^{\ell} \left(\mathbf{F}_{\text{q}}/q\right) \cdot \mathbf{dl} = \int_0^{\ell} \mathbf{v} \times \mathbf{B} \cdot \mathbf{dl} = -\omega r \int_0^{\ell} B_x \, dz = -\omega r \ell \overline{B}_x \tag{1}$$

[I] Internal resistance and other characteristics of batteries, are discussed in I. Buchman, *Batteries in a Portable World* 2nd edition, self-published online: http://www.buchmann.ca/Chap9-page3.asp.

that drives charge in a sense opposite to that of the battery. In the last relation of Eq. (1) I defined the vertically-averaged x-component of the magnetic field

$$\overline{B}_x = \frac{1}{\ell}\int_0^{\ell} B_x\, dz\,, \tag{2}$$

which is an experimentally convenient quantity. Proceeding in the same way, one obtains an emf of the same magnitude and polarity in the left vertical wire.

Application of Kirchhoff's voltage law to the right (or left) current loop yields the relation

$$V - 2iR_{\mathrm{i}} - \left|\mathcal{E}_{ind}\right| = 0 \tag{3}$$

from which follows the current

$$i = \frac{V - \left|\mathcal{E}_{ind}\right|}{2R_i} = \frac{V - \omega r \ell \overline{B}_x}{2R_i}\,. \tag{4}$$

It is now clearer from Eq. (4) why the battery is not shorted out when its terminals are connected by a spinning wire frame. The faster the frame rotates, the larger will be the induced emf, and the smaller will be the current ($2i$) drawn from the battery. Were the frame to rotate close to its theoretical limit, the current drawn from the battery would be close to zero. I will consider shortly how fast the frame can rotate.

In concluding this section, I address briefly the fundamental question of whether Faraday's law of induction has a role to play in producing the induced emf. According to the form of the law ordinarily displayed (in SI units) in textbooks,

$$\mathcal{E} = -\frac{d\Phi}{dt}\,, \tag{5}$$

an emf is induced in a circuit by the time-variation of magnetic flux Φ through the circuit. Ostensibly it would seem that there is *no* time-varying magnetic flux through the *planar* surface of the wire frame in the 'simplest motor'. This is correct. However, when the frame is spinning, each vertical wire sweeps out a differentially small surface area $dS = \ell r\omega dt$ in the time interval dt, to which the field $\overline{B}_x$ is perpendicular. Thus there is a differential change in magnetic flux $d\Phi = \overline{B}_x dS$ resulting in an induced emf of magnitude $d\Phi / dt = \overline{B}_x dS / dt = \omega \ell r \overline{B}_x$. Although it is beyond the scope of this paper to demonstrate, the application of Faraday's law to a circuit with a *moving* boundary, as in the present case, leads to *two* terms: one is equivalent to the time variation of the magnetic field perpendicular to a surface area; the other is equivalent to the Lorentz force law.[4] In the case of our model of the 'simplest motor', the first term vanishes because the magnetic field is time-independent, and the second term remains. *Both* contributions, however, are a consequence of Faraday's law, Eq. (5).

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[4] P. Lorrain and D. Corson, *Electromagnetic Fields and Waves* 2nd Ed. (Freeman, San Francisco, 1970) pp. 339-341.

Feynman has remarked in his famous *Lectures* upon the phenomenon of induction as conceptually unique.

> 'I know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of *two different phenomena*.'[5]

The italics are Feynman's. The two different phenomena to which he referred are described by the Lorentz force law and Faraday's law of induction. It is somewhat difficult to know what to make of Feynman's assertion, for, as the originator of his own form of electrodynamics, he of all people must have been thoroughly conversant with a phenomenon as basic as induction. Nevertheless, Feynman has been wrong before in matters relating to electrodynamics (for example, the case of the Aharonov-Bohm effect in quantum mechanics, in which a magnetic field can affect the quantum interference pattern of a beam of charged particles that does *not pass through* the magnetic field)[6], and I believe that his characterization of the situation is not correct in the present instance. It is *not* the case that one must account for induction in terms of two different phenomena. Both are a consequence of Faraday's law.

Torque and Power—A Second Look

Torque is the first moment of a force—i.e. the cross product of a moment arm and a force. The torque $\mathbf{N}_{\text{mag}}$ arising from the horizontal magnetic forces on the two vertical arms of the frame can be calculated from the expression

$$\mathbf{N}_{\text{mag}} = 2\int \mathbf{R}\times\mathbf{dF} = 2i\int \mathbf{R}\times\left(\mathbf{dl}\times\mathbf{B}\right) \tag{6}$$

in which $\mathbf{R}$ is the coordinate vector of a point on the right vertical wire (the left side having been accounted for by the factor 2) defined with respect to an arbitrarily chosen origin at the center of the neodymium magnet. It is not difficult to evaluate the integral in Eq. (6), but it is simpler[7] and more useful to evaluate the scalar product of the torque with the angular velocity $\boldsymbol{\omega}$, which is directed along $-z$, and thereby obtain the rate at which electrical energy is expended in rotating the wire frame

$$P_{\text{rot}} = \boldsymbol{\omega}\cdot\mathbf{N}_{\text{mag}} = 2i\omega r\ell\overline{B}_x = 2i\left|\mathcal{E}_{\text{ind}}\right|. \tag{7}$$

The rotational power thus turns out to be the product of the total current drawn from the battery and the induced emf.

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[5] R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, 1964) Vol II, page 17-2.

[6] M. P. Silverman, More Than One Mystery: Quantum Interference with Correlated Charged Particles and Magnetic Fields; *American Journal of Physics* **61** (1993) 514-523; M. P. Silverman, *Quantum Superposition: Counterintuitive Consequences of Coherence, Entanglement, and Interference* (Springer, NY, 2008), pp 16-17.

[7] There are two reasons why calculating the power is simpler: (1) The scalar product with the angular velocity projects out only the vertical component of the torque, and (2) one can use the familiar vector identity $\mathbf{A}\cdot\mathbf{B}\times\left(\mathbf{C}\times\mathbf{D}\right)=\left(\mathbf{A}\cdot\mathbf{C}\right)\left(\mathbf{B}\cdot\mathbf{D}\right)-\left(\mathbf{A}\cdot\mathbf{D}\right)\left(\mathbf{B}\cdot\mathbf{C}\right)$ in evaluating the resulting integral. Note that the *x*-component of $\mathbf{R}$ is the constant length r.

With the foregoing result, I can check to see whether electrical energy is conserved in the circuit. The electrical power expended in rotation together with the power dissipated as Joule heating must sum to the power P_{E} provided by the battery

$$P_{\mathrm{E}} = P_{\mathrm{rot}} + (2i)^2 R_{\mathrm{i}}. \tag{8}$$

Upon substituting $P_{\mathrm{E}} = 2iV$ and $P_{\mathrm{rot}} = 2i\left|\mathcal{E}_{\mathrm{ind}}\right| = 2i\omega r\ell\bar{B}_x$ into Eq. (8) and dividing both sides of the equation by 2*i*, one regains the Kirchhoff voltage relation of Eq. (3). Thus, electrical energy is conserved.

However, there is still a loose end to tie. Just as the net force on an object will accelerate the object, thereby continuously increasing its velocity, the application of an unbalanced torque to the wire frame would result in a continuously increasing angular velocity. This does not happen because friction between the lower ends of the spinning frame and the cylindrical surface of the magnet results in a net counter-torque of approximate magnitude

$$N_{\mathrm{f}} = 2fr_{\mathrm{M}} \tag{9}$$

where *f* is the force of friction and r_{M} is the radius of the magnet. The equation of motion of the wire frame is then given by Newton's 2nd law applied to rotation

$$I_{\mathrm{z}}\frac{d\omega}{dt} = N_{\mathrm{mag}} - N_{\mathrm{f}} = 2ir\ell\bar{B}_x - 2fr_{\mathrm{M}} \tag{10}$$

in which I_{z} is the moment of inertia of the wire frame along the z-axis, and *f* is independent of the relative speed between surfaces (and therefore independent of ω). Upon replacing the current *i* by the equivalent expression in Eq. (4), I can recast Eq. (10) in terms of the single variable, angular frequency ω

$$\frac{d\omega}{dt} + \alpha\omega = \beta \tag{11}$$

with constants

$$\alpha = \frac{\left(r\ell\bar{B}_x\right)^2}{I_{\mathrm{z}}R_{\mathrm{i}}} \tag{12}$$

$$\beta = \frac{r\ell\bar{B}_x V}{I_{\mathrm{z}}R_{\mathrm{i}}} - \frac{2fr_{\mathrm{M}}}{I_{\mathrm{z}}}. \tag{13}$$

As a dimensional check of consistency, one can verify that α has the dimension of $(\text{time})^{-1}$ and that β has the dimension of $(\text{time})^{-2}$, in keeping with the dimension of $d\omega/dt$.

Equation (11) is a simple first-order linear differential equation of the kind ordinarily encountered in introductory physics courses (e.g. in the treatment of RC circuits). The solution for a rotor starting at rest at time $t = 0$ is

$$\omega(t) = \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right). \tag{14}$$

Thus, α^{-1} is a characteristic time constant (relaxation time) τ of the system and β/α is the asymptotic (or steady-state) angular frequency ω_∞. Theoretically ω reaches ω_∞ after an infinite amount of time, but practically only a few time constants τ suffice.

Instead of using Eq. (4) to eliminate i from Eq. (10), I can use Eq (1) to eliminate ω, thereby obtaining the following equation for time development of the current

$$\frac{di}{dt}+\frac{i}{\tau}=\frac{i_\mathrm{f}}{\tau} \tag{15}$$

in which

$$\tau=\frac{I_\mathrm{z}R_\mathrm{i}}{\left(r\ell\bar{B}_x\right)^2} \tag{16}$$

is the system relaxation time identified previously, and

$$i_\mathrm{f}=\frac{r_\mathrm{M}}{\mathrm{r}}\frac{f}{\ell\bar{B}_x} \tag{17}$$

is a combination of system parameters with the dimension of electric current. i_f is, in fact, the frictionally-limited asymptotic current, as seen from the solution corresponding to Eq. (14)

$$i(t)=i_\mathrm{f}\left(1-e^{-t/\tau}\right). \tag{18}$$

The steady-state angular frequency ω_∞ and current i_f are related to one another through the expression

$$\omega_\infty=\frac{1}{r\ell\bar{B}_x}\left(V-2R_\mathrm{i}i_\mathrm{f}\right), \tag{19}$$

whose form is seen to be a rearrangement of Kirchhoff's voltage law.

Numerical Details of My Own 'Simplest Motor'

The motor that I put together in a few minutes had the following approximate dimensions: $2r$ = 2.3 cm, ℓ = 6.0 cm , $2r_\mathrm{M}$ = 2.0 cm. The mass of each arm of the wire frame was about 268 mg, leading to an estimated moment of inertia $I_\mathrm{z}=7.1\times10^{-8}\ \mathrm{kg\cdot m^2}$. [Treating the wire frame as two vertical arms of length ℓ connected by a top crosspiece of length $2r$ leads to $I_\mathrm{z}=mr^2\left(1+\frac{r}{3\ell}\right)\sim mr^2$ where m is the mass of the frame.] Using a gauss meter to sample the magnetic field along a vertical arm, I estimated $\bar{B}_x$ at ~ 0.2 Tesla.

I used a moderately fresh AA battery with open-circuit potential measured to be V = 1.501 V and internal resistance measured to be R_i = 1.324 Ω before introduction of the wire frame. A standard procedure for measuring the internal resistance R_i of a battery with a voltmeter is to measure first the open-circuit potential V_0 and then the potential V when the battery is part of a single-loop circuit containing a known resistance R. It is then straightforward to show that $R_i = R\left[\left(V_0/V\right)-1\right]$. The resistance R should not be too large (whereupon too small a current will be drawn) or too low (in which case too high a current will be drawn and the battery will heat up rapidly or even short out).

With the motor assembled, the frame spun stably for long periods of time, and I determined the mean frequency of several trials to be 4.71 Hz by placing a small paper flag on one arm and recording approximately 10 seconds of action with a Canon SD450 digital camera. The camera timer recorded the duration of action precisely, and, by viewing the recording in the slow-playback mode, I could count precisely the number of rotations of the flag. The operating frequency, controlled by friction, is low compared with the maximum steady-state frequency [from Eq. (14) or (19) of $\omega_\infty/2\pi \to V/2\pi r\ell\bar{B}_x$ ~ 1731 Hz in the absence of friction.

From Eq. (1) I determined the induced emf to be $\mathcal{E}_{ind}$ = 4.09 mV, from which it followed from Eq. (4) that i_f = 565 mA. The relaxation time of the motor circuit was determined from Eq. (16) to be τ = 4.9 s. This value is a little high, a consequence, I have ascertained, of the change in battery potential and internal resistance during its service to the motor. Measurements of the open-circuit potential and internal resistance immediately following the end of each recording session yielded the lower values 1.482 V and 0.663 Ω, respectively. Substituted into Eq. (16), the lower resistance leads to a shorter and more realistic relaxation time of 2.5 s. Finally, from Eq. (17), I estimated the frictional force at each end of the wire frame to be $f \sim 7.7\times10^{-3}$ N to within a factor of 2 given the variability in battery operating conditions.

Concluding Remarks

The 'simplest motor in the world' is an example of a free induction motor, operating in accordance with Faraday's law (as all motors do) under the somewhat unusual condition of an unconstrained rotational axis. For a motor with specified geometrical parameters and magnetic field, the time for the rotor to spin up to its steady-state value is controlled by the internal resistance of the battery, an electrical quantity that can vary during operation of the motor. The achievable steady-state frequency, given the same geometry and magnet, is controlled by the internal resistance of the battery and sliding friction between the wire frame and magnet. The lower the friction and internal resistance, the shorter the relaxation time and higher the steady-state frequency.

Watching the frame rotate around the neodymium magnet, an inquisitive person might wonder what would happen if the frame and battery were held fixed but the magnet were sufficiently lightweight and free to turn. Would it (the magnet) rotate? The answer to this question, although deducible immediately from the principle of relativity, is provided in a dramatic fashion by another of the fascinating physics toys I have encountered over the years, and which I usually demonstrate whenever I teach courses on electromagnetism. It is marketed by Andrews Manufacturing under the name *Top Secret*.[8] The toy comprises a black plastic pedestal about 8 centimeters in diameter upon which a silver top can be made to spin practically

[8] "Cool Magnetic Toys You Can Buy", http://www.coolmagnetman.com/magemtoy.htm

interminably (i.e. for days). There is no apparent source of external energy. I will not give away the secret here, except to point out (what any alert experimenter would deduce and confirm quickly) that the spinning top is a cylindrical magnet.